Eur. Phys. J. B **10**, 187–191 (1999)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Competition, efficiency and collective behavior in the "El Farol" bar model

M.A.R. de Cara[a], O. Pla, and F. Guinea

Instituto de Ciencia de Materiales, Consejo Superior de Investigaciones Científicas, 28049 Madrid, Spain

Received 13 November 1998

Abstract. The *El Farol* bar model, proposed to study the dynamics of competition of agents in a variety of contexts (W.B. Arthur, Amer. Econ. Assoc. Pap. Proc. **84**, 406 (1994)) is studied. We characterize in detail the three regions of the phase diagram (efficient, better than random and inefficient) of the simplest version of the model (D. Challet, Y.-C. Zhang, Physica A **246**, 407 (1997)). The efficient region is shown to have a rich structure, which is investigated in some detail. Changes in the payoff function enhance further the tendency of the model towards a wasteful distribution of resources.

PACS. 02.50.-r Probability theory, stochastic processes, and statistics – 02.50.Ga Markov processes – 05.40.-a Fluctuation phenomena, random processes, and Brownian motion

1 Introduction

In recent years there has been a growing interest in understanding the dynamics of systems of interacting individuals with competing goals (frustration). Simple rules for the behavior of the individuals may lead to unexpected properties in the behavior of the collectivity. These rather general premises can apply to problems in different fields, like economy [1], ecology [2] or physics [3].

To illustrate these facts Brian Arthur introduced what he called "*El Farol*" bar problem (EFBP) [4]. N individuals decide, at each time step, to go to a bar or to stay at home. The bar is enjoyable only if the attendance does not surpass some critical number, that can be thought of as some kind of *comfort capacity*. But each individual does not know beforehand what is going to happen. To be able to make the decision for the next time step the individuals (which we will call *agents* in the following, as in previous literature of this model) are provided each one with a set of strategies. Using these strategies, and the knowledge of what has happened in the portion of the history that they can recall, the agents take decisions.

Challet and Zhang [5] have given a precise set of rules which determine the model. The two possible choices, going to the bar or staying at home, are represented by 0 and 1. A choice is successful if the agents which make it are in the minority (comfort capacity = 50%). The outcome of a given simulation is represented by a series of 0's and 1's which characterize the successful choices at each time step. Each agent uses a fixed set of s strategies, taken at random from the pool of all possible strategies. Strategies use the full information of the m previous outcomes to decide the next move. As there are 2^m possible combinations of past events, the number of strategies is 2^{2^m}. After each event, the agents update the score of their set of strategies. The gain made by the successful strategies can either be a fixed constant, or depend on the size of the group formed at that time step. In the simplest version of the model, one point is assigned to each successful strategy. When an agent has two or more strategies with the same score, one of them is picked at random. This choice of payoff is the one discussed in detail below. The model is defined by the three parameters: N, the number of agents, m, the number of time steps used by each strategy in determining the next best move, and s, the number of strategies available to each agent. Extensions to other payoff schemes, similar to those used in [5–7] are also mentioned. Note that the original work [4] used a much less constrained set of strategies and a different comfort capacity (60%).

The model, with the set of rules described above, was investigated in [8,6]. The authors analyze the mean size and the fluctuations of the groups taking each of the two choices available. It is argued that the model can be characterized in terms only of the combination $\rho = 2^m/N$. The average group size is $N/2$. The distribution of sizes is symmetrical around this value. The mean quadratic deviation from the average, σ, is a measure of the number of points accumulated by all the agents. This number is maximum when the two groups are almost equal, in which case $\sigma \sim O(1)$. As function of σ^2/N and ρ three regimes can be distinguished, as function of the total number of strategies at play [8]:

(i) when $\rho \gg 1$, the number of strategies available to the agents is small, and the value of σ approaches the limit expected when the agents take random decisions, $\sigma^2/N = 1/4$;

[a] e-mail: angeles@quijote.icmm.csic.es

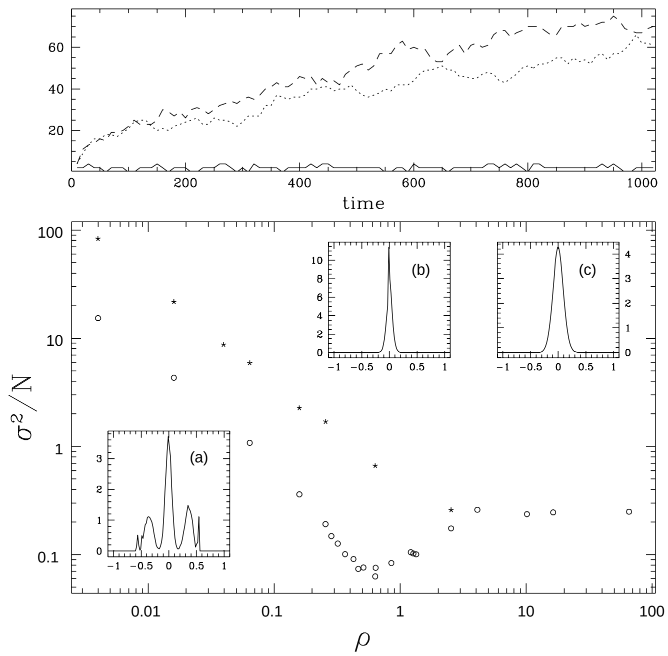

Fig. 1. Different phases found in the EFBP. The lower part shows the evolution of σ^2/N as function of ρ, circles are for $s = 2$ and stars are for $s = 6$. The insets show histograms of the attendance number in the different phases, with $N = 101$: (a) efficient, $m = 2$, $s = 2$; (b) better than random $m = 6$, $s = 2$; and (c) inefficient $m = 10$, $s = 2$. The top figure shows the difference in punctuation between the maximum scored and the minimum scored strategies in these three cases: dotted line for (a), dashed line for (b), and continuous line for case (c).

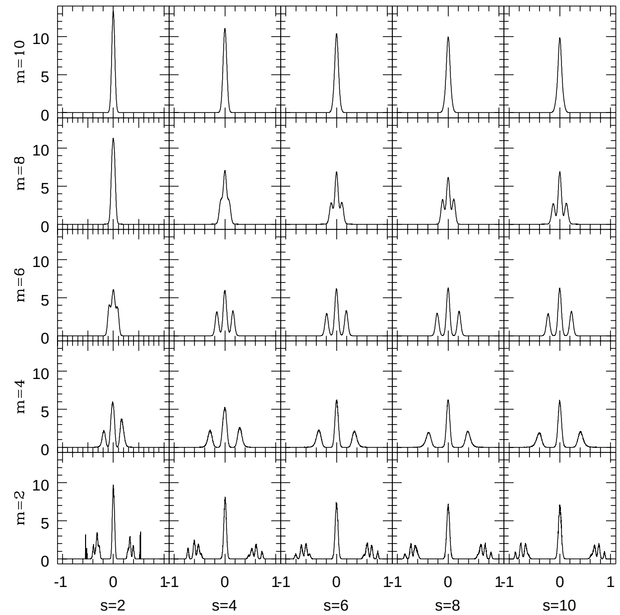

Fig. 2. Attendance numbers distributions for $N = 1001$.

(ii) if $\rho \ll 1$, almost all possible strategies are in possession of the agents, and their performance is worst than random, as $\sigma^2/N > 1/4$;

(iii) finally, for $\rho \sim 1$ the agents perform statistically better than random. The curve of σ^2/N *versus* ρ shows a minimum.

The authors define regime (i) as inefficient, as the agents have little information, and regime (ii) as efficient, as agents have all available information at their disposal.

In Section 2, we analyze the model defined above, with emphasis on the structure shown in the efficient region. Section 3 presents an interpretation of the results. Then, Section 4, we discuss results obtained by varying the payoff function which determines the choice of strategies. Section 5 analyzes a seemingly trivial variation of the model: the majority game, when it becomes preferable to be in the majority. The final section presents the conclusions.

2 Minority game

The transition discussed in [8] is displayed in Figure 1 for $s = 2$ and $s = 6$. The difference between the efficient and inefficient regimes is sharper for small values of s. Each simulation of the model starts from a history of length $m + 3$ to initialize the scores of the strategies. The results shown in the paper are averages over the 2^{m+3} possible initial conditions defined in this way. In almost all cases, the system evolves towards a steady state which is independent of the initial conditions.

The peaks in the size distributions are always well approximated by Gaussian functions. The large value of σ in the efficient region is due to the formation of new peaks away from $N/2$. A pictorial view of this effect is shown in Figure 2, where the different regimes are studied by varying m and s. The attendances have been normalized to one in the interval $[-1, 1]$. In the range of values of ρ where three peaks can be clearly resolved, the weight of the central peak is one half of the total, and the other two peaks include one fourth of the recorded attendances. The central peak is always well approximated by a Gaussian of width $\sqrt{N}/2$ (see also Fig. 3), which corresponds to random choices by the agents.

As one leaves the efficient region, the peaks merge with the central one, whose width decreases first and then increases, to reach the random value for large values of ρ. For small values of ρ, the peak structure is very rich, and seems self similar, as shown in Figure 3.

As pointed out in [8], it is somewhat unexpected the poor performance of the agents when a large amount of information is available. It is even more remarkable the rich structure shown in Figure 3, which shows that the evolution is far from random. This behavior is also consistent with the existence of non trivial patterns in the time series, beyond the reach of the agents [8].

A plot of the attendances at successive times is shown in Figure 4. We have chosen the parameters in such a way that the distribution of attendances shows three separated peaks.

We have completed the study the evolution of the different peaks by analyzing their evolution after an initial series of random choices. In the time series shown

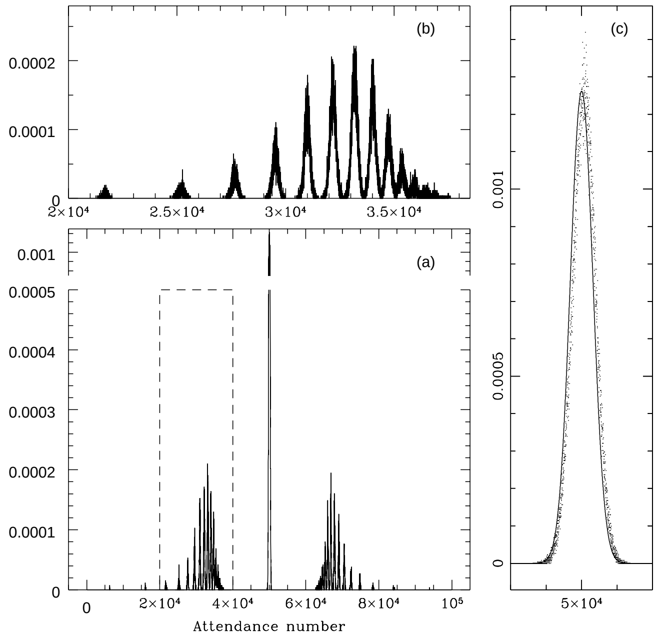

Fig. 3. Attendance numbers distribution for $N = 100001$, $s = 4$, and $m = 4$, normalized in the interval $[0, 100001]$. (a) Full distribution where the y-axis has been truncated in order to appreciate the spreading of the lateral peaks. (b) Magnification of the region marked in (a) with dashed lines. (c) Points in the central peak. The continuous line is a Gaussian, centered at $N/2$, with weight half of the total distribution and deviation $\sqrt{N}/2$.

in Figure 5, the agents make choices randomly, although their strategies keep updating the scores. At a given time step (2048), the agents start to use the strategies at their disposal.

The peak structure is robust, and develops immediately. As shown in Figure 5, the peaks split from the central peak and move to their positions in the steady state discussed earlier.

3 Interpretation

The results presented in the previous section allow us to gain some understanding of the complex dynamics of the efficient regime. In this region, no strategy can stay with the highest score for long. The repeated use of a given strategy by a significant number of agents leads to the raise of other strategies, preferably those more anticorrelated with the one at play. As a result of this, the most punctuated strategy (the best considered by the agents) has many chances of making its users to loose. And, eventually, the agents segregate into anticorrelated groups when some degree of evolution is incorporated [9].

For simplicity, we now assume that there are two anticorrelated strategies, x and $\bar{x}$ which have the highest scores most of the time. Let us denote n_x and $n_{\bar{x}}$ the number of agents which have strategy x and $\bar{x}$. We can

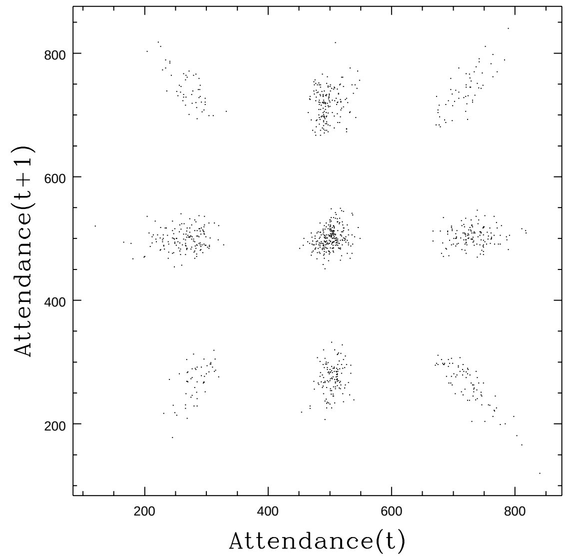

Fig. 4. Attendance in a given group at two successive intervals. The parameters used are $s = 2$, $m = 2$ and $N = 1001$.

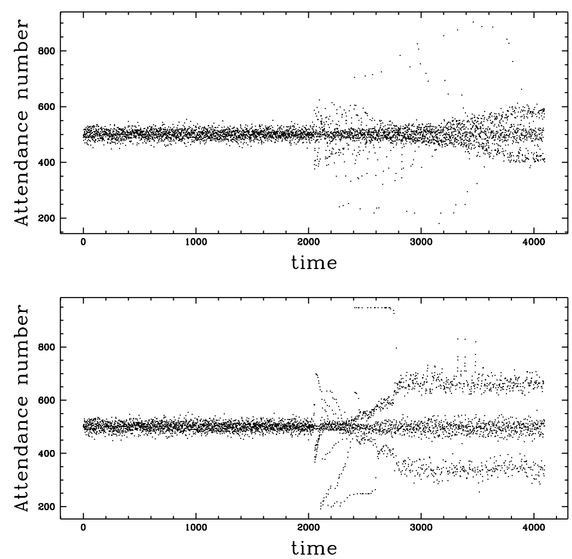

Fig. 5. Attendance number *versus* time for the game in which a transition is forced from a random game to a minority game (see text). The parameters of the minority game are: $N = 1001$, $s = 4$, and $m = 4$ (6) for the bottom (top) graphs.

take $n_x \approx n_{\bar{x}} = n_{\rm correl}$. We now denote as $n_{\rm random}$ the number of agents which have neither x nor $\bar{x}$. The choices of these $n_{\rm random}$ agents can be taken to be at random, as they are unable to recognize the series which give rise to the high scores of x and $\bar{x}$.

When strategy x has the highest score, the two groups will have sizes close to $n_{\rm random}/2 + n_{\rm correl}$ and $n_{\rm random}/2 - n_{\rm correl}$, respectively. This outcome will give no points to x, while strategy $\bar{x}$, which would have lead to the most favorable choice, gains one point. If the score of $\bar{x}$ remains below that of x, the process repeats itself. A steady state is reached when the scores of x and $\bar{x}$ differ by, at most, one point. Then, an outcome with two unequal groups of sizes $n_{\rm random}/2 + n_{\rm correl}$ and $n_{\rm random}/2 - n_{\rm correl}$ is followed by the formation of two groups of similar size, $\approx N/2$. The fact that there are $n_{\rm random}$ agents acting at random implies that these values are the average of Gaussian peaks of similar width.

We can estimate the value of $n_{\rm correl}$ from the analysis in [7]. We classify the 2^{2^m} strategies into 2^m mutually uncorrelated, maximally correlated or anticorrelated classes. Then, $n_x \approx N/2^m = 1/\rho$.

The previous analysis gives a plausible explanation of the three peaks observed throughout most of the efficient region of parameter space. It can be extended, in a straightforward way, to the case when the dominant strategies are more than two. The main new ingredient is that there are situations in which two, or more, dominant strategies can have the same score. Let us imagine that the strategies with the highest scores are $x_1, x_2, \bar{x}_1$ and $\bar{x}_2$. Then, at a given instant, the strategy with the highest score can be x_1, x_2 ..., but also x_1 and x_2 (or similar combinations) simultaneously. If, in addition, x_1 and x_2 lead to the same outcome, the majority group will be of size $n_{\rm random}/2 + n_{x_1} + n_{x_2}$. This combination will be, probably, less likely, leading to lower peaks further away from the average, in agreement with the findings reported here.

We have checked that there is a trivial case where this analysis reproduces the observed evolution: $m = 2$ and $s = 16$, where all agents have all strategies. The attendance histograms show two sharp peaks at 1 and N, and a Gaussian peak with half the weight of the total distribution at $N/2$, and deviation $\sqrt{N}/2$.

4 Varying the rewards for the winners

We now look to the effect of changing the way in which the different strategies are updated after each outcome. The simplest modification is to relate the change in the score to the size of the minority group [5,6]. In the following, we assume that the payoff, Δp, depends linearly with the size, a. If the score is incremented by a, strategies which lead to groups with attendances close to $N/2$ are favored. If, on the other hand, the score is incremented by $N/2 - a$, the tendency is the opposite, and strategies which lead to very small groups are favored.

The distributions generated by these two payoff choices are plotted in Figure 6. The distribution obtained by the step payoff discussed in the previous section is also plotted, for comparison.

Contrary to intuition, the two distributions seem to go in the opposite direction to what the choice of payoff leads to think. It must be noted that, when the second choice of payoff function is shifted by a constant, $\Delta p = N/2 - a + k$, the central peak tends to disappear, and it is replaced by two peaks at the sides. This result is similar to other findings with a payoff which also favors small groups [6].

We interpret the broad structure for the payoff function a as due to the swift shuffle of the highest ranking strategies. Outcomes with nearly equal groups give rise to large changes in the scores of the strategies. Thus, long living cycles, of the type described in the previous section, cannot form. The highest ranking strategy changes rapidly. As all strategies are in play, groups of many sizes are generated, despite the fact that the payoff favors sizes close to $N/2$.

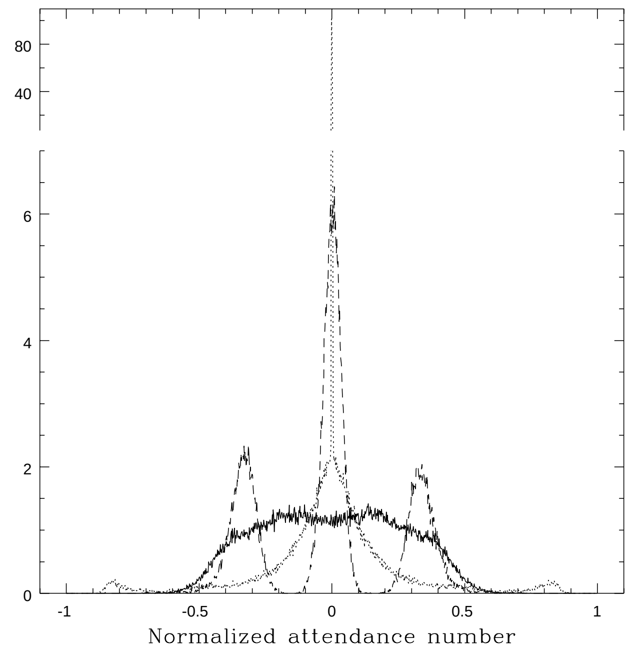

Fig. 6. Attendance distributions for $N = 1001$, $m = 4$, and $s = 4$. Dashed line is for the step payoff, continuous line for $\Delta p = a$, and dotted line for $\Delta p = N/2 - a$.

In the opposite case, with payoff function equal to $N/2 - a$, we ascribe the large peak at $N/2$ to frequent situations when many strategies have the same score. This situation is self sustaining, as, when the two groups are of sizes $N/2$ and $N/2 + 1$, there is no change in the scores of the strategies. This is what happens in half of the possible 2^{m+3} initial conditions, and corresponds to the delta peak in Figure 6. The rest of the distribution is a good average of what happens in the other half of the initial conditions. The shift of the payoff by a constant described earlier reduces the probability of tie-ups, and leads to a double peaked distribution. These peaks displaced from the center seem, in this case, related to the two peaks in the step payoff case. It is likely that the evolution of the model is governed by cycles with a few dominating strategies.

5 Majority game

We have also studied the majority game, in which the agents prefer to be in a overcrowded bar or leave the bar empty. The methodology is the same as in the minority game, in which the different initial conditions tend to give similar results. Here, initial conditions may make big changes in the attendance distributions.

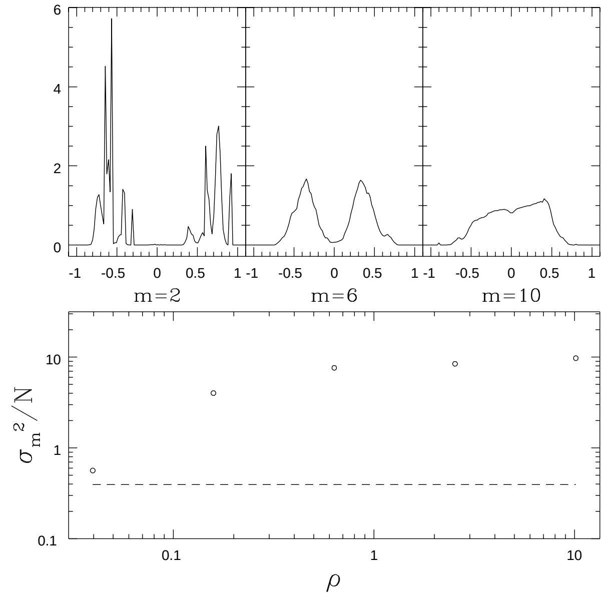

Fig. 7. The analogous diagram of Figure 1 for the majority rule. Here $N = 101$ and $s = 4$. The dashed line is for $\sigma_m = N/2^s$.

Results are trivial (the full majority is attained at all time steps) only when all agents have all strategies ($s = 2^{2^m}$). Even in this case, and depending on the initial conditions, the group (0 or 1) which obtains the majority may oscillate in time.

The obtained distributions for different values of m and, consequently, ρ, are plotted in Figure 7.

The particular placement of the fixed points makes that a more convenient measure of the efficiency should be used. We will use the mean deviation, σ_m, calculated around the value N for the attendance, and shifting the attendances a below $N/2$ to $a + N$. Thus, σ_m also gives a measure of the overall gain made by the agents. In the three plots of attendances, where the attendance axis is not folded, the two large peaks near the limits are not shown. These peaks correspond to limit cycles where the attendances do not fluctuate.

The relative weight of this peak, for $s = 4$, at sufficiently large times, is 0.56 for $m = 2$, 0.078 for $m = 6$ and 0.031 for $m = 10$. The number of agents which are able to coordinate among themselves and take part in this cycle is, on the average, $N - N/2^s$, if $s < 2^{2^m}$. Then, the lower limit for σ_m^2/N is $N/2^{2s}$. This value is also plotted in Figure 7.

The relative weight of this peak, which represents the average number of coordinated agents, converges at sufficiently large times to 0.56 for $m = 2$, 0.078 for $m = 6$ and 0.031 for $m = 10$. It is remarkable that the agents are not too effective in acting in a coordinated manner. Most initial conditions lead to histories where the majority group is well below the intuitive natural limit. This result is consistent with the spin glass features reported in [8].

6 Conclusions

As we have seen, the *El Farol* bar problem has a rich structure. We have focused mostly on the behavior in the efficient regime, where most of the strategies are at the disposal of the agents. As already remarked in [8], the model has many features in common with frustrated systems in statistical mechanics. In particular, most initial conditions lead to a poor performance of the system as a whole. The model seems unable to select a pool of strategies such that the global gain by the agents is maximized. In particular, those agents which have access to the strategies with the highest scores at a given moment perform worse than those which do not. The latter play basically at random, and profit from the unproductive coordination of the players using the nominally best strategies.

This effect seems to remain when the payoff to the different strategies is varied. It is also remarkable that the intrinsic frustration of the model shows up when the agents try to be in the majority. Most initial conditions lead to evolutions where the agents fail to coordinate among themselves.

Financial support from DGCYT through project No. PB96/0875 and the European Union through project ERB4061PL970910 is gratefully acknowledged.

We would like to thank the helpful comments of N. García, Enrique Louis, Pedro Tarazona and Yi-Cheng Zhang.

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